$f(x)=f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)$

4. $\log(1+\sin x) \fallingdotseq x-\frac{x^2}{2}+\frac{x^3}{6}$

$f(x)=\log(1+\sin x)$

$f'(x)=\frac{\cos x}{1+\sin x}$, $f''(x)=\frac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}=\frac{-(1+\sin x)}{(1+\sin x)^2}=\frac{-1}{(1+\sin x)}$

$f'''(x)=\frac{\cos x}{(1+\sin x)^2}$

$f(0)=0,\ f'(0)=1,\ f''(0)=-1,\ f'''(0)=1$

$f(x)=f(0)+\frac{x}{1}f'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{3!}f'''(0)+\cdots$

$=0+x-\frac{x^2}{2}+\frac{x^3}{3!}+\cdots$

$\fallingdotseq x-\frac{x^2}{2}+\frac{x^3}{6}$

5. $\boxed{\frac{x}{e^x-1}\fallingdotseq 1-\frac{x}{2}+\frac{x^2}{12}}$

難問

$\lim_{h\to 0}\frac{e^h-1}{h}=1$

$f(0)=1$

$f'(0)=\frac{1}{2}$

$f(x)=\frac{x}{e^x-1}$, $f'(x)=\frac{e^x-1-x(e^x)}{(e^x-1)^2}=\frac{e^x-1-x\cdot e^x}{(e^x-1)^2}$

$f''(x)=\frac{e^x-e^x-x\cdot e^x}{(e^x-1)^4}=-\frac{-xe^x}{(e^x-1)^4}$

$f'''(x)=\frac{-(e^x+xe^x)(e^x-1)^4+xe^x\cdot 4(e^x-1)^3\cdot e^x}{(e^x-1)^8}$

$\log(z+1)=\log e^x=x$

$z=e^x-1$ とおくと $x=\log_e(z+1)$ $x\to 0,\ z\to 0$

$f(x)=\frac{\log_e(z+1)}{z}=\log_e(z+1)^{\frac{1}{z}}\to 0\ (z\to 0)$

$f'(x)=\frac{dy}{dz}\cdot\frac{dz}{dx}=\left[\frac{\frac{z}{z+1}-\log_e(z+1)}{z^2}\right]e^x=\left\{\frac{1}{z(z+1)}-\left(\frac{1}{z^2}\right)\log_e(z+1)\right\}e^x$

$=\left[\frac{1}{(e^x-1)e^x}-\left(\frac{1}{(e^x-1)^2}\log_e e^x\right)\right]e^x=\frac{1}{e^x-1}-\frac{x\cdot e^x}{(e^x-1)^2}$

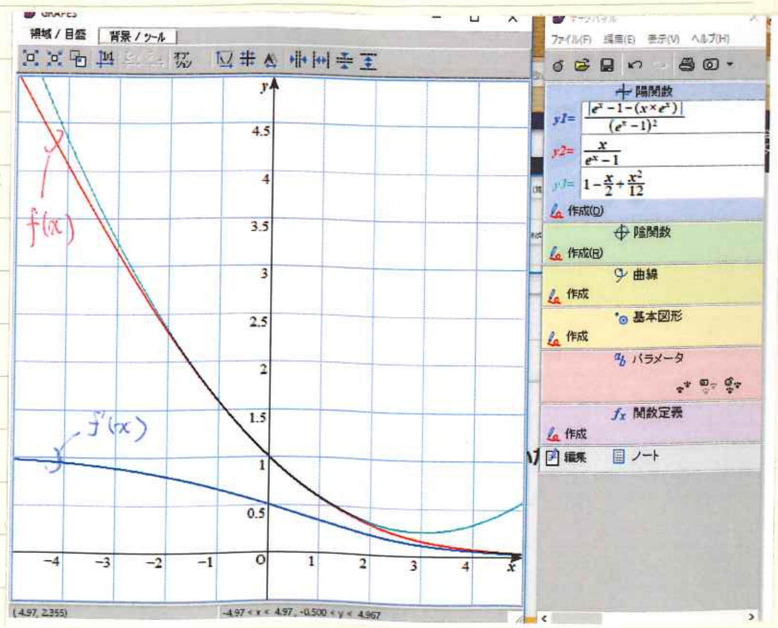
$=\frac{1}{z}-\frac{(z+1)\cdot\log(z+1)}{z^2}$

末

$\frac{(e^x-1)(1-x)-x}{(e^x-1)^2}=\frac{(1-x)}{e^x-1}-\frac{x}{(e^x-1)^2}$

$=\frac{1}{e^x-1}-\frac{xe^x-x}{(e^x-1)^2}-\frac{x}{(e^x-1)^2}$

$=\frac{1}{e^x-1}-\frac{xe^x}{(e^x-1)^2}$



$\log_a x^M=y$

~H30.9.7.

6. $f(x)=(1+x)^{\frac{1}{x}}\fallingdotseq e\left(1-\frac{x}{2}\right)$

$f'(x)=\frac{1}{x}(1+x)^{\frac{1}{x}-1}$

$g(x)=\log(1+x)^{\frac{1}{x}}=\frac{1}{x}\log(1+x)$

$g'(x)=\frac{1}{(1+x)^{\frac{1}{x}}}\times\frac{1}{x}(1+x)^{\frac{1}{x}-1}=\frac{1}{x}(1+x)^{-1}=\frac{1}{x(1+x)}$

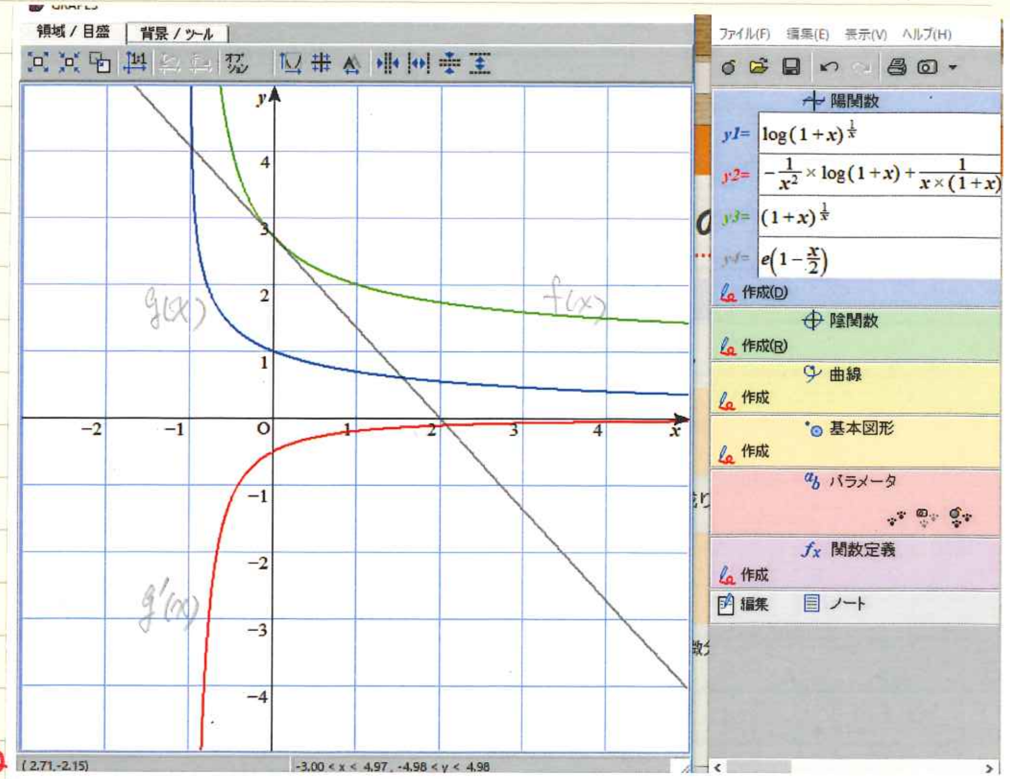
$=\left[\frac{1}{x}\log(1+x)\right]'=-\frac{1}{x^2}\log(1+x)+\frac{1}{x}\frac{1}{1+x}=\frac{-(1+x)\log(1+x)+x}{x^2(1+x)}$

$h(x)=(1+x)^{\frac{1}{x}}$

$h(0)=1$

$h'(x)=\frac{1}{x}(1+x)^{\frac{1}{x}-1}$

難問



マクローリン

分母だけにTalor展開する

$g(x)=e^x-1$ とすると

$g(0)=0$

$g'(x)=e^x$ $g'(0)=1$ $g''(x)=e^x$ $g''(0)=1$

$\therefore g(x)=x+\frac{1}{2!}x^2+\frac{1}{3!}x^3+\cdots$

$-\frac{1}{6}+\frac{1}{4}=\frac{-2+3}{12}$

$f(x)=\frac{x}{e^x-1}=\frac{1}{1+\frac{1}{2!}x+\frac{1}{3!}x^2+\frac{1}{4!}x^3+\cdots}$ $z=\frac{1}{2!}x+\frac{1}{3!}x^2+\frac{1}{4!}x^3+\cdots$

$=\frac{1}{1+z}=h(z)$

$h(0)=1$. $h'(z)=-(1+z)^{-2}$ $h''(z)=2(1+z)^{-3}$ $h'''=-2\cdot 3(1+z)^{-4}$ H30.9.7 !!

$h'(0)=-1$ $h''(0)=2$ $h'''(0)=-6$

$h(z)=1-z+z^2-z^3+\cdots$ $f(x)=1-\left(\frac{1}{2!}x+\frac{1}{3!}x^2+\frac{1}{4!}x^3\cdots\right)+\left(\frac{1}{2!}x+\frac{1}{3!}x^2\cdots\right)^2$

$\fallingdotseq 1-\frac{x}{2}+\frac{1}{12}x^2$